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Efficient Sensor Position Selection Using Graph Connectivity

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Abstract: The traffic control has to be managed systematically to avoid the traffic congestion especially in the busy city. The efficient and systematic traffic control is studied in this research using the connectivity and compatibility graphs of traffic intersections. From the graphs drawn, the most efficient route can be determined and the capacity of traffic flow can be maximized by finding the minimum number of edges or the minimum number of vertices. In this paper, the 4-ways intersection stream at Jalan Membunga Machang is chosen to find suitable locations to place sensors that are used to collect traffic data. From the graphs obtained, there are 12 vertices identified where 36 edges were connected to it. By using the algorithm of graph theory, four sets of minimal edge control were determined and these edges were validated using the MAPLE software.

Keywords: Graph Theory, Sensor, Traffic Light.

1 Introduction

The purpose to have traffic signals is to regulate the conflicts of traffic flows at intersections. If the traffic signal is managed systematically, it can gives benefit to the road users and if it configured incorrectly, the problem associated with it such as traffic congestion will happen. In the traffic light system, the three colors red, green and yellow are chosen to represent different actions. Red is chosen as the color for stop because red has for centuries been used to indicate danger while green is chosen as the color to go because it has the color of nature and can represents safety [1].

The traffic control system was developed significantly since the first non-electric traffic light control was installed in London in 1868 [2]. In 1914, the electric traffic light was developed in Cleveland, Ohio with the red and green as the chosen color schemes for the traffic light. In 1960, the very first traffic controller system was then developed and was introduced in Toronto.

The electromechanical devices were used to perform the traffic control at first [3] and was later upgraded to the Intelligent Transportation System (ITS). ITS was extensively used in urban areas to control traffic at an intersection. ITS is an information and communication technology that aims to provide the infrastructure and vehicles management to improve safety, mobility and efficiency from the basic management system to monitor applications and to more advanced applications that integrate live data and feedback from other sources. Wireless communication, computational technologies and sensing technologies are some of the components implemented in ITS. The accurate traffic data from the traffic flow can be collected by using the semi conductor-based controller known as sensor. These sensors were placed at the intersections to collect the necessary traffic information for traffic control system [2].

Graph theory is the branch of mathematics concerned with networks of points connected by lines (vertices that are connected by the edges). This theory is capable to solve the system of traffic lights as applied on konigsberg bridges by Leonard Euler in 1736. The traffic system developed for this bridge was divided into four parts and connected by seven bridges. The assumption made by Euler for the developed system was everyone will only cross the bridge once [4].

Traffic management is important to regulate the flow of traffic. Congestions can occur due to the increase of urbanization, population growth and the number of vehicles on the road. Based on the report released by The Malaysian Automative Association (MAA), 666,465 vehicles were registered for the year 2014, increase by 1.6% from the previous year. Malaysia also recorded as the third highest car ownership globally with 93% of the households owned personal cars and 54% owned multiple car ownership [5].

The increase in the number of vehicles on the road will definitely create the traffic congestion especially in the cities area. This phenomena will cause the increase in the traffic accidents, environmental and air pollutions, the travel time and the fuel consumptions.

2 Literature Review

The traffic lights was used to regulate the flow of traffic to avoid collisions. The arrangement of the traffic must be managed properly to avoid traffic congestions especially in the cities area where the number of vehicles on the road was doubled due to the increase in population growth. In the research done by [6], the signed graph was used to model the transportation problem where the nodes represents the destination and the edges represents the relationship among them. Based on their analysis, positive (+) sign was obtained and shown the balance graph. The result indicates that the traffic flow at the intersection was smooth and well planned.

As investigated by [7], one of the problem affect the development of most city areas are traffic congestion. This situation makes people late to work, the deliveries cannot be on time and cost a lot of money for fuel expenses. Constructing new roads can be one of the alternative to solve the issue but the cost incurred will be higher and the problem cannot be solved in a short time period. The only solution to this problem is to use the current road network efficiently. They introduced the compatibility graph corresponding to the problem and circular arc graphs to reduce the waiting time at junctions.

In the research done by [8], the interval graph was used to overcome the problem at an intersection among the traffic users. Since the traffic control at an intersection is crucial due to its complexity, the relations among the streams was firstly identified. The streams were considered as vertices and will be connected by an edge if the two streams are able to move simultaneously without any conflicts. The interval graph was obtained and be used to find the optimal number of feasible green light assignments.

Graph Coloring Theory was used by [9] on the intersection of five fork roads. The compatible graph found in this research is not the interval graph. Therefore, the several edges were opened by the researchers to create an interval graph from the subgroup of the compatible graph. Linear programming was also used in their research to optimize the control time for each road in the intersection.

Traffic control problem at an intersection can also be modelled as a graph. From the research done by [3], the cut-set of graph was used to identify the efficient route from the minimum number of edges. The cut-set graph is also known as an edge control set of graph. This graph is obtained by joining the traffic streams that is compatible at the intersection with an edge. The streams which cannot move simultaneously will not be connected to an edge. Minimal edge control represents the flow of traffic at the intersection and can be found by applying the algorithm. Therefore, by finding the minimal edge control set, the sensors can be placed in the right place and the waiting time of traffic users can also be minimized

The compatibility graph was used by [10] by connecting two streams with an edge. These two streams are compatible if they can move simultaneously without any accidents when the green lights are on. The duration for the traffic light to turn to a green light corresponds to an arc of a circle was considered in their research. The spanning subgraph (intersection graph from the compatibility graph) with the most maximal clique used to identify the optimum waiting time for all streams in a cycle [7].

The compatibility graph was also used by [2] in their research. They divide the cliques of the compatible graph as signal groups by taking into consideration the compatible streams that have the green light simultaneously. The spanning subgraph which is the intersection graph corresponding to any feasible green light assignment was formed to calculate the waiting time at the intersection.

3 Methodology

A Identify the Traffic Streams

In this paper, the stream at Jalan Membunga, Machang was considered. The traffic streams were redrawn based on the information obtained from Jabatan Kerja Raya (JKR) Machang as shown in Figure 1.

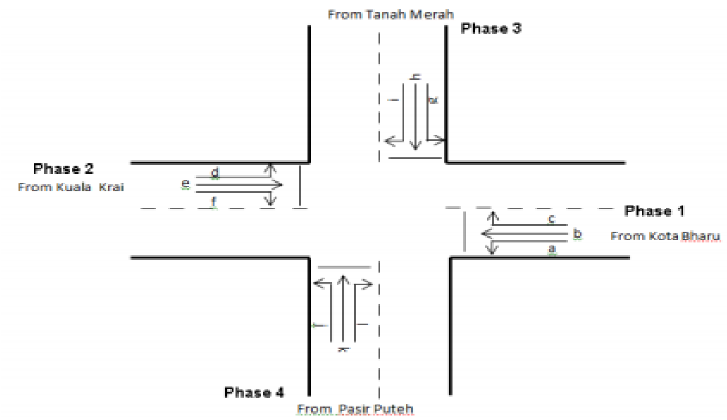

Figure 1: 12 Streams at the 4-way Intersections of Jalan Membunga, Machang, Kelantan.

B Develop the Compatibility Graph

Compatibility graph is a graph *G* whose vertex set, *V* consist of the traffic streams and *E* is the two vertices of *G* are connected if, and only if, the corresponding streams are compatible [10]. Thus, Compatibility graph of *G = (V,E).* A matrix of incidence *A* was form by

$$a_{ij} = \begin{cases} 1, \text{ if movements } k \text{ and } j \text{ do not intersect} \\ 0, \text{ if movements } k \text{ and } j \text{ intersect} \end{cases}$$

where *k* and *j* both are the traffic streams [11]. Hence, the matrix formed is used to draw the compatibility graph by using the software MAPLE.

C Determine the Minimal Edge Control Set

A graph *G* is a pair of sets (V, E), where *V* is the set of edges of *G*. An edge control set *F* is said to be minimal if any proper subset of *F* is not an edge control set of the graph *G*, Niky and Arun [6]. As the edge control set of a graph is not unique, therefore it is important to find the set with the minimum number of edges. Niky and Arun [5] found that the algorithm can be used in order to find the minimal edge control set by using the defnition as follows:

Definition 1: Graph $G = (V,E)$ is considered to be a graph, let H be a subgraph of G and $e \in E(H)$. We define: $C_H(e) = \{e\} \cup \{d \in E(H): d$ is a cut edge of $H - \{e\}\}$
Then, $C_H(e)$ is called the control of e in H.

Algorithm 1: A graph G is considered and a subset $F \subseteq E(G)$ is constructed by the following steps:

Step 1: Let $F := \phi$ and $H := G$
Step 2: While $E(H) \neq \phi$, select any edge $e \in E(H)$
$F := F \cup \{e\}$
$H := H - C_H(e)$
Then, F is the minimal edge control set of the graph G.

D Draw the Connectivity Graph

According to [2] the connectivity graph is a graph where the removal of edges disconnect the compatibility graph. There are two ways to obtain connectivity graph either by removing vertex or removing edges. In this case study, removal of edges based on the compatibility edges is chosen. So, the connectivity graph can be obtained by declaring vertices and edges of the graph. The edges are obtained by subtracting the edges of compatibility graph with edges of minimal control set where the vertices are same.

4 Results and Discussion

The results obtained in this paper are shown in this section. From the compatibility graph drawn, 12 vertices and 36 edges are found. The minimal edge control set is identified to find the effective location to place the sensor from the connectivity graph drawn.

A Compatibility Graph

The compatibility graph shown in Figure 2 is drawn based on matrix A in Figure 3. This graph has 12 labeled vertices and 36 edges. The vertices are connected to each other if they are compatible. This compatibility graph is used to find the minimal edge control to draw the connectivity graph.

$A =$

	a	b	c	d	e	f	g	h	i	j	k	l
a	0	1	1	1	1	0	1	0	1	1	1	1
b	1	0	1	1	1	0	1	0	0	0	0	0
c	1	1	0	0	0	0	1	0	0	1	0	0
d	1	1	0	0	1	1	1	1	1	1	0	1
e	1	1	0	1	0	1	0	0	0	1	0	0
f	0	0	0	1	1	0	1	0	0	1	0	0
g	1	1	1	1	0	1	0	1	1	1	1	0
h	0	0	0	1	0	0	1	0	1	1	1	0
i	1	0	0	1	0	0	1	1	0	0	0	0
j	1	0	1	1	1	1	1	1	0	0	1	1
k	1	0	0	0	0	0	1	1	0	1	0	1
l	1	0	0	1	0	0	0	0	0	1	1	0

Figure 2: Matrix A

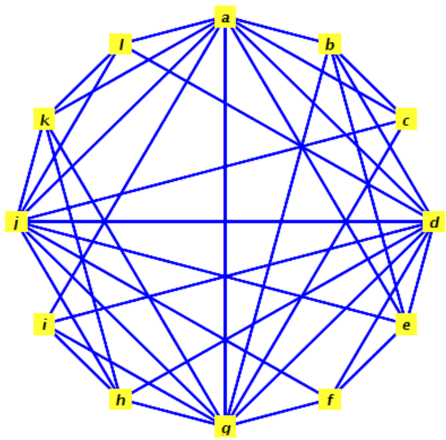

Figure 3: Compatibility Graph

B Minimal Edge Control Set

The set *F = {(a, c), (b, c), (c, g), (c, j)}* is the set of minimal edge control of the compatibility graph. From the set *F* we have *(a,c), (b,c), (c,g), (c,j)* as the four minimal edges control of the compatibility graph. These edges will determine the location to place the sensors. Based on the result obtained, the sensors should be placed on the first, third and fourth phase at the traffic light streams. Noticed that the vertex *c* in the set *F* is the deleted edge in the compatibility graph and as a result, the connectivity graph can be formed.

From set *F*, the edge connectivity of *(a,c), (b,c), (c,g),* and *(c,j)* were proven by using the "IsCutSet(S)" command in Maple to validate whether the sets obtained is true or false. To prove this, the edges that have the smallest degree of vertex is identified which are *(a,b), (b,c), (c,j),* and *(c,g).* These edges are then tested in "IsCutSet(S)" of Maple Software to prove for its connectivity. The result turned to be true for all the four edges.

"IsConnected" command of Maple Software is used to determine whether the graph is connected. From the Graph *G*, the connectivity graph obtained by deleting the disconnected edges using the "DeleteEdge" command. The "IsConnected" command was then used to test the connectivity of the graph after removing the disconnected edges. If the result displayed as true, the graph obtained is proven connected.

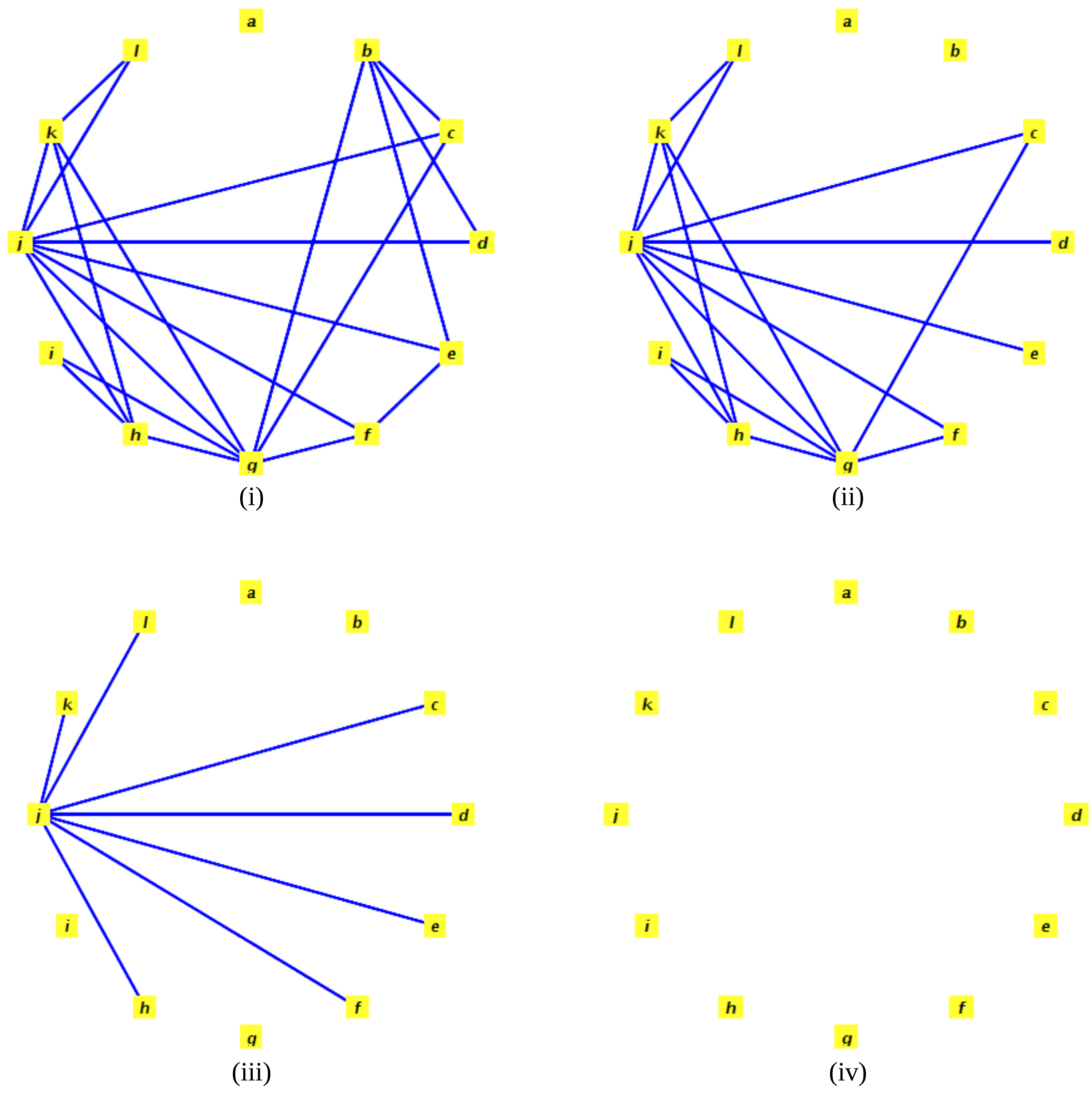

Figure 4: Steps in Deleting the Disconnected Edges using "DeleteEdge" Command

C Connectivity Graph

The graph is not connected because of vertex *c*. And the minimal control edges *(a,c), (b,c), (c,g),* and *(c,j)* obtained from the previous section are the deleted edges. Therefore, the four minimal control edges were removed to get the connectivity graph as shown in Figure 5.

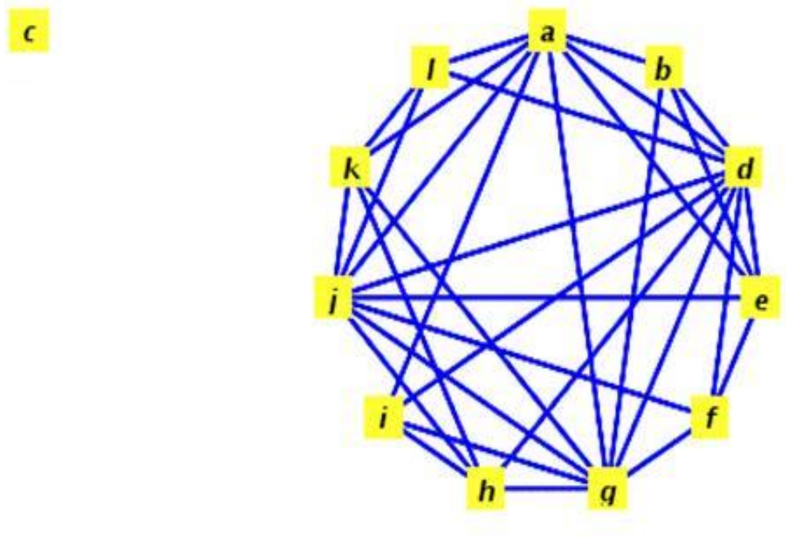

Figure 5: Graph Connectivity

To get the connectivity graph, there are five possible number of traffic streams need to be chosen from 8, 9, 10, 11, and 12 streams. The edge connectivity of 8 streams is zero, one edge is connected for 9 streams, two edges connected for 10 traffic streams and three connected edges formed from 11 traffic streams. In this paper, 12 traffic streams is considered for the 4-way intersection at Jalan Membunga that was controlled by four phases of traffic light.

5 Conclusion

In this paper, the cut-set graph was used to study traffic control problems at an intersection. The four sets of minimal edge control *(a,c), (b,c), (c,g), (c,j)* were identified. The locations to place the sensors were also determined namely phase 1, phase 3 and phase 4 and these places are the most effective places to collect the traffic data.

The research of traffic control in Jalan Membunga, Machang is relevant since the pedestrian road was also considered in this research.

It is recommended to consider the minimum waiting time at the intersection using interval intersection graph for the future research.

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References

[1] Badon, M. (2018, June 14). Red Light, Green Light: The Invention of the Traffic signal. Retrieved from http://designobserver.com/article.php?id=8627.

[2] Arun, K. B., and Niky, B. (2013). Intersection Graph in Traffic Control Problems. International Journal Of Mathematics and Computer Applications Research, 03 (01), 265-270.

[3] Niky, B., and Arun, K. B. (2012a). On a traffic control problem using cut-set of graph. International Journal Advanced Networking and Applications, 03 (04), 1240-1244.

[4] Dooren, P. V. (2009, August 10). Graph Theory and Applications. Retrieved from www.hamilton.ie/ollie/Downloads/Graph.pdf

[5] Nelson. (2014, April 16). Car Ownership in M'sia Third Highest in The World: Nielsen Retrieved from www.thestar.com.my/business/business-news/2014/04/16/car-ownership-in-msia-third-highest-in-the-world

[6] Niky, B., and Arun, K. B. (2012b). On Signed Graph in Traffic Control Problems. International Journal of Combinatorial Graph theory Applications, 05 (01), 57-65.

[7] Darshankumar, D., and Natyangini, J. (2014). Application of Graph Theory in Traffic Management. International Journal of Engineering and Innovative Technology, 03 (12), np.

[8] Niky, B., and Arun, K. B. (2013). On a Traffic Control Problem using Interval Graph. International Global Research Analysis, 02 (08), 2277-8160.

[9] Ziyang, L., Cai, Y., Jixin, C., and Lurong, W. (2011, July 26-28). Intelligent control over the crossing traffic lights by graph theory. In Fuzzy Systems and Knowledge Discovery (FSKD), 2011 Eighth International Conference on (Vol. 3, p. 1377-1380). doi: 10.1109/FSKD.2011.6019900.

[10] Hosseini, S. M., and Orooji, H. (2009). Phasing of Traffic Light at a Road Junction. Applied Mathematical Science, 3 (30), 1487-1492.

[11] Husain, P., Ivan, M. and Ante, M. (2000). Selection of Optimal Traffic Signal Cycle using Graphs. Science in Traffic Preliminary Communication, Vol. 12, 2000, No.1, 67-73.